Estimation of air-gun array signatures from near-gun measurements - least-squares inversion, bubble motion and error analysis

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Summary

We reformulate the usual approach to estimation of air-gun signatures from near-gun records as a least-squares inversion. We show that this has advantages compared to the commonly-used iterative method; we are able to more accurately treat the motion of the air bubble from each gun as the bubble moves away from the hydrophones, we are able to relax the constraint in the current method that the number of recording hydrophones must equal the number of guns in an array, and we are also able to relax the constraint that hydrophones must be placed near to a gun. We show by a singular value analysis, however, that when additional hydrophones are deployed the derived signatures are likely to be most accurate when the hydrophones are close to the guns (e.g. not in a mini-streamer or similar arrangement). Example directional far-field signatures derived by our approach compare well with modeled signatures and are suitable for use in processes such as shot-by-shot signature deconvolution.

Introduction

The general problem of estimating the far-field signature of marine air-gun arrays has received renewed interest lately, primarily because of a desire to take advantage of the improved signal to noise ratio of modern acquisition systems through processing that extends the useable range of seismic frequencies (Williams and Pollatos, 2012). Examples relating to the problem of signature estimation are directional signature deconvolution (Poole et al, 2013) and directional source deghosting (Telling et al, 2014).

In the usual approach, based on work by Ziolkowski et al (1982) and Parkes et al (1984), hydrophones are placed close to each gun in the array, and recordings from the hydrophones are used to derive a “notional signature” for each gun. This is the signature of the gun when other guns of the array are firing and the bubble from each gun is influenced by the pressure field generated by the other guns. Once notional signatures have been derived for a particular shot they can be used to construct the far-field signatures at any desired direction away from the array.

The hydrophones record a contribution from all the guns of the array, but the notional signature of each gun can be obtained by an iterative inversion of the recordings providing there are as many hydrophones as there are guns in the array. Additional hydrophones, if present, can be used to QC the signature estimation (Laws, 2000) but are not directly useable in the estimation itself. The iterative updating also relies on the hydrophone being close to the gun; Parkes (1984) comments that if this is not the case the iteration is unworkable.

There are problematic aspects of the method when it is extended to take account of the motion of the air bubble from each gun, which Parkes (1984) shows is necessary for accurate signature estimates. It is relatively easy to include time-varying distance and arrival times in the formulation to account for propagation from a moving bubble, although this somewhat complicates the iterative solution. A more difficult issue, however, is that the bubble can move away from the hydrophone that it is closest to when the gun fires and then be closer to a different hydrophone. This can lead to instability in the solution, as we will see later, and Yang et al (2010) attempt to deal with this problem by changing the hydrophone used for updating as the iteration progresses. This aspect of the bubble motion affects the signature estimate at later times, and can influence the low frequency content of the estimated signatures.

In this paper we present an alternative least-squares formulation that can more accurately account for the relative motion between the air-gun bubble and the recording hydrophone. Our approach removes the constraint on the number of recording hydrophones and guns, and in principle can be extended to arbitrary field configurations, removing also the requirement for the recording hydrophone to be positioned close to the gun. However, not all field configurations are equal, and not all provide accurate signature estimates, as we show by a singular value analysis of alternative acquisition scenarios.

Time-domain iterative solution

In Ziolkowski et al’s 1982 method, hydrophones are placed near to each gun and record the arrivals from both the nearby gun and also from other guns of the array. The hydrophones are in the near-field of the array, but providing they are 1m or more away they are in the far-field of the bubble from each gun, in the sense that the arrivals at the hydrophones can be described by the standard acoustic wave equation. For each hydrophone i the response to the direct arrivals from the guns is:

$$d_i(t) = \sum_j \frac{1}{s_{i,j}} m_j\left(t - \tau_{i,j}\right)$$

where $m_j(t)$ is the notional signature for the gun at position j, $\tau_{i,j}$ is the delay from gun j to the hydrophone at position i, and $s_{i,j}$ is the distance from gun j to hydrophone i. Further terms can be included in the summation to represent the ghost arrival, the water-bottom reflection, and water-bottom multiples, using assumed or derived sea-surface and water-bottom reflection-coefficients and sea-bottom geometry.

Estimates for the notional signatures can be obtained by the iteration proposed by Parkes (1984), as also discussed in Ziolkowski (1998). An initial estimate of the notional signature $m_i(t)$ of gun i at time t is obtained from $d_i(t)$, the response at the hydrophone nearest to the gun. This plus estimates of other gun signatures for earlier times, available from previous computation, are used to compute an estimate of $d_i(t)$. Then $m_i(t)$ is updated from the discrepancy between the observed and the estimated value of $d_i(t)$, and the procedure repeated for subsequent iterations.

Frequency-domain methods

If for the time being we ignore the motion of the air-gun bubble with respect to the recording hydrophones then in the frequency domain the summation above can be replaced by the linear model **D** = **A M** where **D** is a column vector whose elements D_i are the data from each hydrophone i at a particular frequency, and **M** is a column vector whose elements M_j are the notional signature for each gun j at that frequency. Elements a_{ij} of array **A** contains phase-shift and scaling terms equivalent to the time-shift and scaling terms in the time-domain formulation; for the direct arrival

$$d_i(t) = \sum_j \frac{1}{s_{i,j}} m_j\left(t - \tau_{i,j}\right)$$

and similar additional terms can be included to represent the contributions from the ghost and water-bottom arrivals.

An iterative frequency-domain solution for **M** is possible by a method equivalent to the time-domain iteration already discussed, providing the number of hydrophones equals the number of guns. If the number of hydrophones is greater than the number of guns, a least-squares solution can be obtained from the normal equations $\mathbf{A}^+\mathbf{A}\ \mathbf{M} = \mathbf{A}^+\ \mathbf{D}$, where $\mathbf{A}^+$ is the adjoint of **A** (the complex transpose in this context). This is the approach used by Amundsen (1993, 2000) to derive signatures from a mini-streamer towed beneath the source array, and Amundsen justifies neglecting the bubble motion by asserting that it is not significant at the typical separation between the streamer and the array. Ziolkowski and Johnston (1997) are critical of his approach, however, partly because it neglects the bubble motion but also because of possible positioning errors and contamination of the recordings by the water-bottom reflection. A further issue, as we will see later, is that noise in the streamer records can limit the spatial resolution of the derived notional signatures.

Incorporation of bubble motion

The bubble motion changes both the amplitude and time of the arrivals at the hydrophones. Of these, the amplitude effects are the most significant, whilst the timing effects are relatively small. If we assume that the bubble moves away from a hydrophone at a velocity of 3 m/s then the time of the arrival will increase by 2ms one second after the guns fire, a phase change of one quarter of a period at 125 Hz. Adopting the rule of thumb that phase changes of less than a quarter of a period are not significant, we can say that even one second after the guns fire there is a negligible change in frequencies below 125 Hz.

The amplitude changes are more significant, particularly for the arrivals from a nearby gun. For a hydrophone at 1 m from a gun port, the distance from the bubble to the hydrophone changes from 1m to say 4m one second after the gun fires, and the amplitude of the arrival is then ¼ of its original value. This is a significant change that must be allowed for in the notional signature computation. The effect is generally much smaller, however, for the ghost and water-bottom arrivals, which are also weaker and contribute less to the hydrophone recordings.

We choose, therefore, to neglect the effect of the bubble motion on arrival times and also (although this is not essential for our method) on the amplitude of the ghost or water-bottom arrivals. We can then do forward modeling from notional signatures to hydrophone data by a time-frequency approach, in which phase shifts for arrival times and the scaling of the ghost and water-bottom are applied in the frequency domain and the amplitude scaling for the direct arrival is applied in the time domain. Conceptually we have **D** = **S A' M**, where **D** now represents the hydrophone data at all frequencies and **M** the signatures at all frequencies. **A'** represents the process of using time-constant travel-path lengths to apply phase shifts for the travel times of the direct, ghost and water-bottom arrival, and for the scaling for the ghost and water-bottom arrivals. **S** represents the process of inverse Fourier transformation of the direct arrival from each gun, multiplication by the inverse of the time-varying distance $s_{i,j}(t)$ between gun i and bubble j, and then forward Fourier transformation.

If the number of hydrophones equals the number of guns, notional signatures can again be derived by an iterative approach similar to the time-domain method. An alternative least-squares formulation is $(\mathbf{S\ A'})^+(\mathbf{S\ A'})\ \mathbf{M} = (\mathbf{S\ A'})^+\mathbf{D}$, where $(\mathbf{S\ A'})^+$ is the adjoint of the forward operator, and the form of the adjoint follows from the derivation of this equation. Figure 1 shows directional far-field signatures that were obtained from a conjugate-gradients solution of this expression. Signatures were derived for angles of 0, 30 and 60 degrees from the vertical and filtered to 0 – 125 Hz;

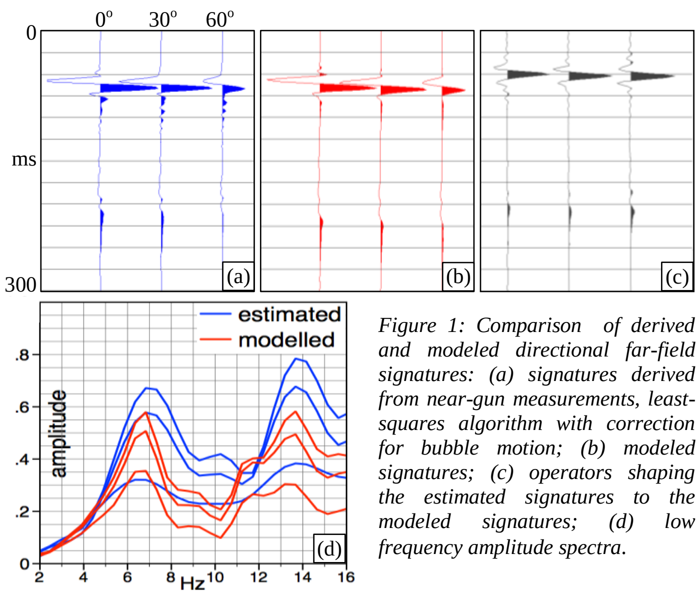

Figure 1: Comparison of derived and modeled directional far-field signatures: (a) signatures derived from near-gun measurements, least-squares algorithm with correction for bubble motion; (b) modeled signatures; (c) operators shaping the estimated signatures to the modeled signatures; (d) low frequency amplitude spectra.

also shown are equivalent modeled signatures from the Nucleus™ modeling package (derived using a combination of modeling and measured signatures). Figure 1(c) shows the filter that shapes from the estimated to the modeled signature, as a convenient way of summarising their difference. The sharp, symmetric, initial pulse of the shaping filter suggests that the signatures have comparable amplitude and phase spectra; the features seen at later times in the filters suggest a slight difference in the residual bubble content. This may be the reason for the slightly higher amplitude of the estimated signatures at low frequencies seen in Figure 1(d).

There are advantages to the least-squares approach compared to the time-domain iterative solution, which derive from using the adjoint operator to move from data to model space as opposed to updating via the nearest neighbor. The adjoint simultaneously updates the signature estimates for all of the guns using all of the data, irrespective of the relative number of guns and hydrophones, and irrespective of their location. Additionally, if a bubble moves away from a hydrophone and towards another then this change is implicit in the formulation of the adjoint providing that the supplied bubble velocity is sufficiently accurate. The issue that Yang et al (2010) address by changing the hydrophone used for updating is dealt with automatically by our algorithm and no further change is required to account for this effect.

We can demonstrate this last point by examining the stability of the solutions when updating by either the adjoint or nearest neighbor. Figure 2 shows that when bubble motion is not included in the iterative frequency-domain algorithm the nearest-neighbor updating is stable, but becomes unstable when bubble motion is included. Updating by the adjoint is stable, however, in all cases.

Alternative recording geometries

A least-squares approach provides the possibility of using alternative recording geometries with varying numbers and placements for the hydrophones. However, some configurations will produce more reliable signature estimates than others. The standard configuration, in which the hydrophone is attached to or is near to the gun body, has advantages in terms of the stability of the recording geometry and perhaps also the signal to noise content of the recordings. Given the possibility of deploying additional hydrophones, what is the best way to do that?

We investigated this question by a singular value analysis of the forward modeling matrix, without bubble motion but incorporating both the direct and the ghost arrivals. When singular value decomposition is applied to a linear model it produces a simplified relationship $\mathbf{Y} = \mathbf{\Lambda X} + \mathbf{N}$ between the model and the data, where $\mathbf{\Lambda}$ is a diagonal matrix of singular values, **Y** and **N** are representations of the data and noise in the data space and **X** is a representation of the model in the model space. Since $\mathbf{\Lambda}$ is diagonal the relationship between each data and model component is simply $y_i = \lambda_i x_i + n_i$, where λ_i is the i'th singular value. An estimate of model component x_i can be obtained from $y_i / \lambda_i = x_i + n_i / \lambda_i$, although this contains a noise component n_i / λ_i which increases in size as λ_i decreases. It is therefore the relative size of the singular values that determines the reliability of the model components; estimated components in directions with smaller singular values will contain larger errors due to noise.

We investigated the reliability of notional signatures from gun recordings for a source array consisting of 3 strings with 6 guns in each string and a gun depth of 7m. We

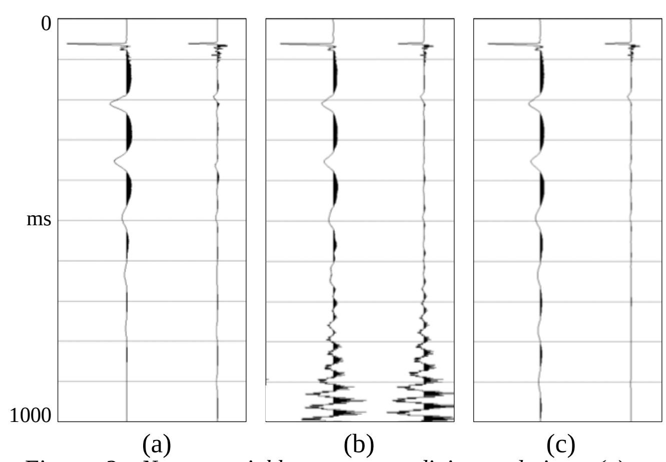

Figure 2: Nearest-neighbor versus adjoint updating: (a) no bubble motion, nearest-neighbor updating - stable. (b) bubble motion, v_x = 2m/s, v_y = -2m/s, nearest-neighbor updating - unstable. (d) bubble motion, adjoint updating - stable.

started by placing recording hydrophones 1m above each gun as in the standard configuration and computed singular values for the forward modeling matrix at 125 Hz and 1 Hz. These are the blue lines shown in Figure 3.

With some basic assumptions about the behavior of the noise, the signal to noise power ratio in each model component can be assumed to equal $1/\lambda_i^2$ times the signal to noise ratio of the equivalent data component. If we assume for example a signal to noise ratio of 10% in the recorded data then for the lower singular values in the figure, below 0.7, the signal to noise in the equivalent model components will be 20% or greater. From examination of the model eigenvectors, these components have more rapid variations from gun to gun ("higher spatial frequencies") compared to the components with larger singular values. Array configurations where the gun volumes in each string change rapidly from gun to gun will be less accurately estimated than arrays whose gun volumes vary more slowly. Furthermore, singular values at 1Hz are generally slightly smaller than those at 125Hz; where there is error in the notional signature estimates it will be slightly larger at lower frequencies.

An alternative to using near-gun hydrophones that was mentioned earlier is to obtain observations on a mini-streamer deployed beneath the array. In Figure 3 the purple lines are the singular values for notional signature estimates from a mini-streamer with 48 hydrophones at 1 m separation, centered beneath the array and towed at a depth of 20 m. Only the first few singular values are significantly greater than 0, implying that the mini-streamer gives reliable estimates only for the low spatial frequency components of the signatures. The streamer records do not provide accurate information on the spatial variation of the gun signatures, which is critical for the derivation of directional far-field signatures.

One way to improve the reliability of the standard configuration is to simply increase the number of data points. The black lines in Figure 3 shows the singular values computed for a configuration in which additional hydrophones are placed 1m below each gun. The singular values are higher in all cases compared to the original version of the standard configuration, implying a reduced signal to noise ratio for the notional signature estimates.

Conclusions

The usual approach to deriving signatures from near-gun recordings fits well with the physics and geometry of the problem; the recordings are dominated by the arrival from the nearby gun, and other weaker arrivals are from earlier and hence already computed portions of the gun signatures. The bubble motion complicates the solution, however, as

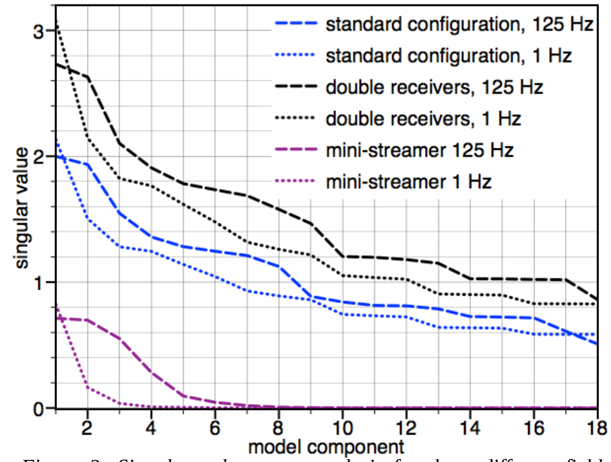

Figure 3: Singular value error analysis for three different field configurations; blue is the standard configuration with a hydrophone 1m above each gun, black is the standard configuration with an additional hydrophone 1m below each gun, purple is a mini-streamer deployed 13 m below the array. Small singular values for the higher model components indicate decreased reliability when gun volumes vary rapidly throughout the

we have seen, and if not correctly accounted for may cause errors in the signatures at later times and lower frequencies. This could be a concern for broadband processing if the derived signatures are used for signature deconvolution.

Inclusion of the bubble motion leads to a slightly awkward formulation for the forward modeling from signatures to hydrophone records, but with some assumptions about the relative importance of the bubble amplitude and timing effects the problem can be expressed as a hybrid time-frequency modeling from the signatures to the data. Providing the bubble velocity is known sufficiently accurately it is inherent in this approach that it deals with the movement of the bubble away from a hydrophone and towards another, which is a problematic aspect of the usual approach. A least-squares solution allows, at least in principle, an increased number of hydrophones to be used, and providing these are placed close to the guns this can help to increase the reliability of the derived signatures.

A topic for further study is how the accuracy of the derived signature estimates is impacted by errors in the assumed array geometry, or in other parameters of the problem such as the sea-surface reflection coefficient or assumed bubble velocity. Our time-frequency modeling could also readily be used for an investigation of this subject.

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